Questions

Q1.

Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) P(A' | B')

(2)

(b) Explain why the events *A* and *B* are not independent.

(1)

The event *C* has P(*C*) = 0.20

The events *A* and *C* are mutually exclusive and the events *B* and *C* are statistically independent.

(c) Draw a Venn diagram to illustrate the events *A*, *B* and *C*, giving the probabilities for each region.

(5)

(d) Find P([$B \cup C$]′)

(2)

(Total for question = 10 marks)

Q2.

Three bags, *A*, *B* and *C*, each contain 1 red marble and some green marbles.

Bag *A* contains 1 red marble and 9 green marbles only
Bag *B* contains 1 red marble and 4 green marbles only
Bag *C* contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from bag *A*.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag *B*.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from bag *C*.

(a) Draw a tree diagram to represent this information.

(2)

(b) Find the probability that Sasha selects 3 green marbles.

(2)

(c) Find the probability that Sasha selects at least 1 marble of each colour.

(2)

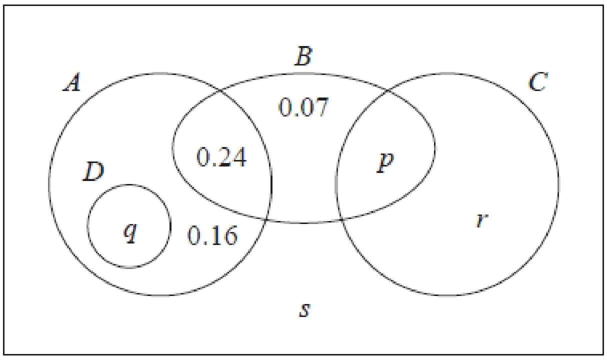
(d) Given that Sasha selects a red marble, find the probability that he selects it from bag *B*.

(2)

(Total for question = 8 marks)

Q3.

The Venn diagram shows the probabilities associated with four events, A, B, C and D



(a) Write down any pair of mutually exclusive events from A, B, C and D

(1)

Given that P(B) = 0.4

(b) find the value of p

(1)

Given also that A and B are independent

(c) find the value of q

(2)

Given further that P(B' | C) = 0.64

(d) find

(i) the value of r
(ii) the value of s

(4)

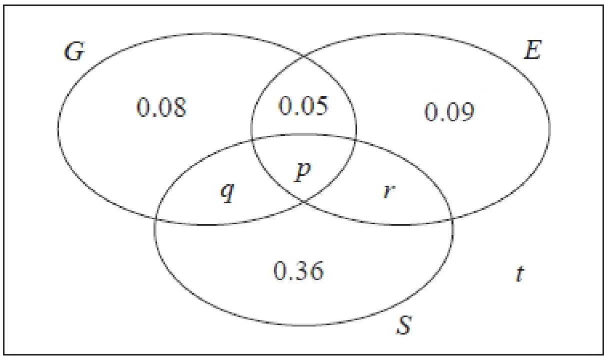
(Total for question = 8 marks)

Q4.

A large college produces three magazines.
One magazine is about green issues, one is about equality and one is about sports.
A student at the college is selected at random and the events G, E and S are defined as follows

G is the event that the student reads the magazine about green issues
E is the event that the student reads the magazine about equality
S is the event that the student reads the magazine about sports

The Venn diagram, where p, q, r and t are probabilities, gives the probability for each subset.



(a) Find the proportion of students in the college who read exactly one of these magazines.

(1)

No students read all three magazines and P(G) = 0.25

(b) Find

(i) the value of p
(ii) the value of q

(3)

Given that $P(S \mid E) = \frac{5}{12}$

(c) find

(i) the value of r
(ii) the value of t

(4)

(d) Determine whether or not the events $(S \cap E')$ and G are independent.

Show your working clearly.

(3)

(Total for question = 11 marks)

Q5.

The heights of females from a country are normally distributed with

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

Given that 1% of females from this country are shorter than k cm,

(a) find the value of k

(2)

(b) Find the proportion of females from this country with heights between 150 cm and 175 cm

(1)

A female, from this country, is chosen at random from those with heights between 150 cm and 175 cm

(c) Find the probability that her height is more than 160 cm

(4)

The heights of females from a different country are normally distributed with a standard deviation of 7.4 cm

Mia believes that the mean height of females from this country is less than 166.5 cm

Mia takes a random sample of 50 females from this country and finds the mean of her sample is 164.6 cm

(d) Carry out a suitable test to assess Mia's belief.

You should

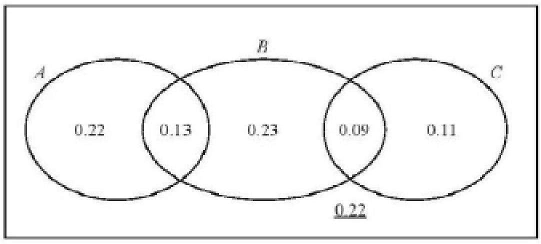
- state your hypotheses clearly
- use a 5% level of significance

(4)

(Total for question = 11 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$\mathrm{P}(A' \mid B') = \dfrac{\mathrm{P}(A' \cap B')}{\mathrm{P}(B')}$ or $\dfrac{0.33}{0.55}$	M1	3.1a
	$= \dfrac{3}{5}$ or 0.6	A1	1.1b
		(2)	
(b)	e.g. $\mathrm{P}(A) \times \mathrm{P}(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq \mathrm{P}(A \cap B) = 0.13 = \frac{52}{400}$ or $\mathrm{P}(A' \mid B') = 0.6 \neq \mathrm{P}(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
		(5)	
(d)	$\mathrm{P}(B \cup C)' = 0.22 + 0.22$ or $1-[0.56]$ or $1-[0.13+0.23+0.09+0.11]$ o.e.	M1	1.1b
	$= \mathbf{0.44}$	A1	1.1b
		(2)	
		(10 marks)	

Notes:

(a)

M1: for a correct ratio of probabilities formula and at least one correct value.

A1: a correct answer

(b)

for a fully correct explanation: correct probabilities and correct comparisons.

(c)

B1: for box with B intersecting A and C but C not intersecting A.(Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$)No box is B0

M1: for method for finding P($B \cap C$)

A1: for 0.09

M1: for 0.13 and their 0.09 in correct places and method for their 0.23

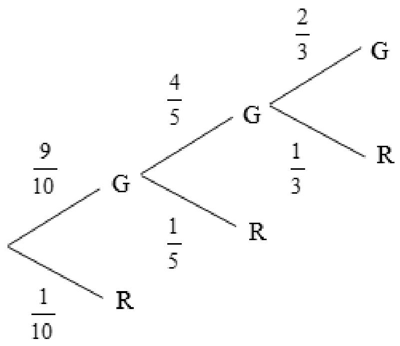
A1: fully correct

(d)

M1: for a correct expression – ft their probabilities from their Venn diagram.

A1: cao

Q2.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		B1	This mark is given for a correct shape and labels for a tree diagram
		B1	This mark is given for the correct probabilities shown
(b)	$\frac{9}{10}\times\frac{4}{5}\times\frac{2}{3}$	M1	This mark is given for a multiplication of three probabilities
	$=\frac{12}{25}$	A1	This mark is given for the correct probability that Sasha selects three marbles
(c)	$\frac{9}{10}\times\frac{1}{5}+\frac{4}{5}\times\frac{1}{3}$	M1	This mark is given for the addition of two products
	$=\frac{21}{50}$	A1	This mark is given for the correct probability that Sasha selects at least one marble of each colour
(d)	P(red form *B* | red selected) = $\dfrac{\frac{9}{10}\times\frac{1}{5}}{1-\frac{12}{25}}=\frac{9}{50}\times\frac{25}{13}$	M1	This mark is given for determining the correct ratio of probabilities
	$=\frac{9}{26}$	A1	This mark is given for the correct probability that Sasha selects a red marble from bag *B*
			(Total 8 marks)

Q3.

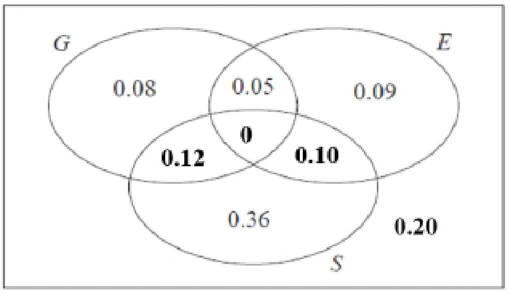
	Scheme	Marks	AO
(a)	A, C or D, B or D, C	B1 **(1)**	1.2
(b)	$[p = 0.4 - 0.07 - 0.24 =]$ $\underline{\mathbf{0.09}}$	B1 **(1)**	1.1b
(c)	A and B independent implies		1.1b
	$\mathrm{P}(A)\times 0.4 = 0.24$ or $(q+0.16+0.24)\times 0.4 = 0.24$	M1	
	so $\mathrm{P}(A) = 0.6$ and $q = \underline{\mathbf{0.20}}$	A1cso **(2)**	1.1b
(d)(i)	$\mathrm{P}(B' \mid C) = 0.64$ gives $\dfrac{r}{r+p} = 0.64$ or $\dfrac{r}{r+\text{"0.09"}} = 0.64$	M1	3.1a
	$r = 0.64r + 0.64$ "p" so $0.36r = 0.0576$ so $r = \underline{\mathbf{0.16}}$	A1	1.1b
(ii)	Using sum of probabilities = 1 e.g. "0.6" + 0.07 + "0.25" + s =1	M1	1.1b
	so $s = \underline{\mathbf{0.08}}$	A1 **(4)**	1.1b
		(8 marks)	

	Notes
(a)	B1 for one correct pair. If more than one pair they must all be correct. Condone in a correct probability statement such as $\mathrm{P}(A\cap C) = 0$ or correct use of set notation e.g. $A\cap C = \varnothing$ BUT e.g. "P(A) and P(C) are mutually exclusive" alone is B0
(b)	B1 for $p = 0.09$ (Maybe stated in Venn Diagram [VD]) [If values in VD and text conflict, take text or a value <u>used</u> in a later part]
(c)	M1 for a correct equation in one variable for P(A) or q using independence <u>or</u> for seeing **both** $\mathrm{P}(A\cap B) = \mathrm{P}(A)\times\mathrm{P}(B)$ <u>and</u> $0.24 = 0.6\times 0.4$
	A1cso for $q = 0.20$ or exact equivalent (dep on correct use of independence)
Beware	Use of $\mathrm{P}(A) = 1 - \mathrm{P}(B) = 0.6$ leading to $q = 0.2$ scores M0A0
(d)(i)	1st M1 for use of $\mathrm{P}(B' \mid C) = 0.64$ leading to a correct equation in r and possibly p. Can ft their p provided $0 < p < 1$
	1st A1 for $r = 0.16$ or exact equivalent
(ii)	2nd M1 for use of total probability = 1 to form a linear equation in s. Allow p, q, r etc Can follow through their values provided each of p, q, r are in [0, 1)
	2nd A1 for $s = 0.08$ or exact equivalent

Q4.

	Scheme	Marks	AO
(a)	$0.08 + 0.09 + 0.36 = \underline{0.53}$	B1	1.1b
		(1)	
(b)(i)	$\left[\text{P}(G \cap E \cap S) = 0 \ \Rightarrow\right] \ \underline{p = 0}$	B1	1.1b
(ii)	$[\text{P}(G) = 0.25 \ \Rightarrow] \ 0.08 + 0.05 + q + "p" = 0.25$	M1	1.1b
	$\underline{q = 0.12}$	A1	1.1b
		(3)	
(c)(i)	$\left[\text{P}(S \mid E) = \frac{5}{12} \ \Rightarrow\right] \frac{r + "p"}{r + "p" + 0.09 + 0.05} = \frac{5}{12}$	M1 A1ft	3.1a 1.1b
	$\left[12r = 5r + 5 \times 0.14 \ \Rightarrow\right] \ \underline{r = 0.10}$	A1	1.1b
(ii)	$\left[0.08 + 0.05 + "0.12" + "0" + 0.09 + "0.10" + 0.36 + t = 1 \ \Rightarrow\right] \ \underline{t = 0.20}$	B1ft	1.1b
		(4)	
(d)	$\text{P}(S \cap E') = 0.36 + "q" \ [= 0.48]$	B1ft	1.1b
	$\text{P}\left(\left[(S \cap E')\right] \cap G\right) = "q" [= 0.12]$ **and** $\text{P}(G) = 0.25$ **and** $\text{P}(S \cap E') \times \text{P}(G) = "0.48" \times \frac{1}{4}$ or 0.12	M1	2.1
	$\text{P}(S \cap E') \times \text{P}(G) = 0.12 = \text{P}\left(\left[(S \cap E')\right] \cap G\right)$ **so are independent**	A1	2.2a
		(3)	
		(11 marks)	

	Notes
(a)	B1 for 0.53 (or exact equivalent) [Allow 53%]
(b)(i)	B1 for $p = 0$ (may be placed in Venn diagram)
(ii)	M1 for a linear equation for q (ft letter "p" or their value if $0 \leqslant p \leqslant 0.12$) $\Rightarrow$ by $p + q = 0.12$
	A1 for $q = 0.12$ (may be placed in Venn diagram)
(c)(i)	M1 for a ratio of probabilities (r on num and den) (on LHS) with num < den and num <u>or</u> den correct ft. Allow ft of letter "p" <u>or</u> their p where $0 \leqslant p < 0.86$ but "+ 0" is not required.
	1st A1ft for a correct ratio of probabilities (on LHS) allowing ft of their p where $0 \leqslant p < 0.86$
	2nd A1 for $r = 0.1(0)$ or exact equivalent (may be in Venn diagram) **Ans only 3/3**
(ii)	B1ft for $t = 0.2(0)$ (o.e.) <u>or</u> correct ft i.e. $0.42 - (p + q + r)$ where p, q, r and t are all probs
(d)	B1ft for $\text{P}(S \cap E') = 0.48$ (with label) (ft letter "q" or their value if $0 \leqslant q \leqslant 0.12$)
	M1 for attempting all required probs (labelled) <u>and</u> using them in a correct test (allow ft of q)
	A1 for all probs correct and a correct deduction (no ft deduction here)
SC	**No "P" If correct argument seen apart from P for probability for all 3 marks, award (B0M1A1)**
	If unsure about an attempt using conditional probabilities, please send to review.



Q5.

	Scheme	Marks	AO
(a)	$\left[\text{Let } F \sim \text{N}(166.5, 6.1^2)\right]$ $\text{P}(F < k) = 0.01 \Rightarrow \dfrac{k - 166.5}{6.1} = -2.3263$	M1	3.4
	$k = 152.309...$ $\underline{152}$ or awrt $\underline{152.3}$	A1	1.1b
		(2)	
(b)	$[\text{P}(150 < F < 175) =]$ 0.914840... awrt $\underline{0.915}$	B1	1.1b
		(1)	
(c)	$\text{P}(F > 160 \mid 150 < F < 175)$	M1	3.1b
	$= \dfrac{\text{P}(160 < F < 175)}{\text{P}(150 < F < 175)}$ or $\dfrac{\text{P}(160 < F < 175)}{\text{"(b)"}}$	M1	1.1b
	$= \dfrac{0.7749487...}{\text{"}0.91484...\text{"}}$	A1ft	1.1b
	$= 0.84708...$ awrt $\underline{0.847}$	A1	1.1b
		(4)	
(d)	$\text{H}_0 : \mu = 166.5$ $\text{H}_1 : \mu < 166.5$	B1	2.5
	[Let X = height of female from 2nd country] $\bar{X} \sim \text{N}\left(166.5, \left(\dfrac{7.4}{\sqrt{50}}\right)^2\right)$	M1	3.3
	$\text{P}(\bar{X} < 164.6) = 0.03472...$	A1	3.4
	[0.0347... < 0.05 so significant or reject H_0] There is evidence to support Mia's belief	dA1	2.2b
		(4)	
		(11 marks)	

	Notes
(a)	M1 for standardising (allow $\pm$) with k, 166.5 and 6.1 and set equal to a z value $2.3 < \lvert z \rvert < 2.4$
	A1 for 152 or awrt 152.3 **Ans only** 2/2 [Condone poor use of notation e.g. $\text{P}(\frac{k-166.5}{6.1}) = -2.3263$]
	Allow percentages instead of probabilities throughout.
(b)	B1 for awrt 0.915
(c)	1st M1 for interpreting demand as an appropriate conditional probability ($\Rightarrow$ by 2nd M1)
	2nd M1 for correct ratio of expressions (can ft their (b) on denominator) ($\Rightarrow$ by 1st A1ft)
	1st A1ft for a correct ratio of probs (can ft their "0.9148..." to 3sf from (b) if > 0.775)
	2nd A1 for awrt 0.847
(d)	B1 for both correct hypotheses in terms of μ
	1st M1 for selecting the correct model (needn't use $\bar{X}$ $\Rightarrow$ by standardisation or 1st A1)
	1st A1 for correct use of the correct model i.e. awrt 0.035 (allow 0.04 if $\text{P}(\text{"}\bar{X}\text{"} < 164.6)$ seen)
	Condone $\text{P}(\text{"}\bar{X}\text{"} > 164.6) = 0.9652$ or awrt 0.97 <u>only if</u> comparison with 0.95 is made
ALT	**Use of z value:** Need to see $Z = -1.8(15...)$ **and** cv of $\pm$ 1.6449 (allow 1.64 or better) for 1st A1
ALT	**Use of CR or CV for $\bar{X}$:** Need to see "$\bar{X}$"< 164.7786... or CV = ... (awrt 164.8) for 1st A1
	Condone truncation i.e 164.7 or better
	2nd dA1 (**dep on M1A1** only) for a correct inference in context.
	Must mention <u>Mia's belief</u> **or** <u>mean height of females/women</u>
	Do NOT award if contradictory statements about hypotheses made e.g. "not sig"
SC	**M0** for $\bar{X}$ ~N(**164.6**, ...) If they achieve p = awrt 0.035 (o.e. with z-value or CV of 166.3) **and** a correct conclusion in context is given score M0A0A1 [and SC for awrt 0.97 > 0.95 case]